# Chapter 2
# Syntax and Symbolization

# 2.1 – The Syntax of SL

- *Sentential Logic* or SL: branch of symbolic deductive logic in which sentences are the basic units of logical analysis.
- The *syntax* of a language specifies the basic expressions of a language and the rules that determine which combination of those expressions count as sentences of the language.
- It does not specify what those expressions mean; this is a matter for *semantics*
- When we talk about a language we call that language the *object language*, e.g., SL is our object language.
- A *metalanguage* is a language used to discuss or describe some object language.
- The distinction is relative: we can talk about German in English and vice versa.

# 2.1 – The Syntax of SL

- We employ words in different ways: *use* and *mention*.
- E.g.,

'Minnesota was the 32nd state admitted to the Union' *uses* the word 'Minnesota' to designate a political subdivision.

'"Minnesota" is an Indian word' *mentions* the word 'Minnesota'

- We note the difference between use and mention by using '…' when we mention a word.
- E.g., 'Bob' has three letters

# 2.1 – The Syntax of SL

- We use *metavariables* to talk about the expressions *of the object language SL*: ***P***, ***Q***, ***R***…
- E.g., instead of saying:
  - If '~(H ∨ I)' is an expression of SL consisting of a tilde followed by a sentence of SL, then '~(H ∨ I)' is a negation we may say:
  - If ***P*** is an expression of SL consisting of a tilde followed by a sentence of SL, then ~***P*** is a negation.
  - This sentence is not about ***P***, but of any value of ***P***.

# 2.1 – The Syntax of SL

- Our language SL consists of its *vocabulary* and its *grammar*.
- In our vocabulary, we have
  - *Sentence letters* [A, B, C…],
  - *Sentential connectives* [~, &, ∨, ≡, ⊃],
  - *Punctuation marks* [( ), [ ]].
- Other expressions of SL are formed by *defining* the sentences of SL by:

# 2.1 – The Syntax of SL

*Defn*: Recursive definition of a *sentence* of SL

1) Every sentence letter is a sentence of SL.
2) If ***P*** is a sentence, then ~***P*** is a sentence.
3) If ***P*** and ***Q*** are sentences, then (***P*** & ***Q***) is a sentence.
4) If ***P*** and ***Q*** are sentences, then (***P*** ∨ ***Q***) is a sentence.
5) If ***P*** and ***Q*** are sentences, then (***P*** ⊃ ***Q***) is a sentence.
6) If ***P*** and ***Q*** are sentences, then (***P*** ≡ ***Q***) is a sentence.
7) Nothing is a sentence unless it can be formed by repeated application of 1-6.

- ***Note***: This definition *must* be in terms of **P**, **Q…** (not **p**, **q…**) since it is a definition *in SL*

# 2.1 – The Syntax of SL

- This definition provides an *effective method* of determining whether an expression is a sentence. We can determine, in a finite number of steps, whether or not an expression is a sentence.
- E.g., ‘(~B & (~B ∨ A))’ is a sentence:

‘A’, ‘B’ are sentences from (1.)

‘~B’ is a sentence from (2.)

‘~B ∨ A’ is a sentence from (4.)

‘(~B & (~B ∨ A))’ is a sentence from (3.).

# 2.1 – The Syntax of SL

- A *syntactical* study of a language is a study of the expressions and their relations without regard to possible interpretations.
- A *semantic* study of a language is the study of the possible interpretations for expressions, e.g., truth tables provide a semantics for the connectives of SL.

# 2.1 – The Syntax of SL

More syntactic concepts:

- *Atomic sentence*: ***P*** contains no connectives and has no sentential components.
- *Negation*: The main connective of ~***P*** is '~' and ***P*** is the immediate component.
- *Conjunction*: the main connective of (***P*** & ***Q***) is '&' and ***P*** and ***Q*** are the immediate components.
- *Disjunction*: the main connective of (***P*** v ***Q***) is 'v' and ***P*** and ***Q*** are the immediate components.
- *Material Conditional*: the main connective of (***P*** ⊃ ***Q***) is '⊃' and ***P*** and ***Q*** are the immediate components.
- *Material Biconditional*: the main connective of (***P*** ≡ ***Q***) is '≡' and ***P*** and ***Q*** are the immediate components.

# 2.1 – The Syntax of SL

- *Sentential components* of a sentence: the sentence, its immediate components, and the components of its immediate components.
- *Atomic components* of a sentence: all the components that are atomic sentences.

# 2.2 – Introduction to Symbolization

- *Important*: we are dealing only with sentences of SL that have truth-values, i.e., are either true or false.
- One can link sentences together by way of sentential connectives and parentheses.

# 2.2 – Introduction to Symbolization

*Defn*: A sentential connective is used *truth-functionally iff* it is used to generate a compound sentence from one or more sentences in such a way that the truth-value of the generated compound sentence is *wholly determined* by the truth-values of the sentences from which the compound is generated.

- E.g., the sentential connective 'and' is used truth-functionally in "Dogs bark and cats meow." The truth-value of the sentence is determined by the truth-value of its constituent sentences.

# 2.2 – Introduction to Symbolization

- In SL one uses capital letters to abbreviate sentences: A, B, C …
- In SL capital letters represent *atomic* sentences.
- In SL sentences made up of atomic sentences and sentential connectives are called *compound* sentences.
- In SL we abbreviate 'and' by '&',

*e.g.*, "Socrates is wise and Aristotle is crafty" is symbolized as W & C.

# 2.2 – Introduction to Symbolization

- A sentence of the form ***P*** & ***Q***, where ***P*** and ***Q*** are sentences of SL, is called a *conjunction*, of which ***P*** and **Q** are *conjuncts*.
- A conjunction is true iff both of its conjuncts are true:

| ***P*** | ***Q*** | ***P*** & ***Q*** |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

- This *truth table* is the *characteristic truth-table* for conjunction; it *defines the use* of '&'.

# 2.2 – Introduction to Symbolization

- In SL we abbreviate ‘or’ by ‘$\vee$’,

e.g., “Henry James was a psychologist or William James was a psychologist” is symbolized as $H \vee W$.

- A sentence of the form $\boldsymbol{P} \vee \boldsymbol{Q}$, where $\boldsymbol{P}$ and $\boldsymbol{Q}$ are sentences of SL, is a *disjunction*, of which $\boldsymbol{P}$ and $\boldsymbol{Q}$ are *disjuncts*.

# 2.2 – Introduction to Symbolization

- A *disjunction* is true iff at least one of its disjuncts is true:

| ***P Q*** | ***P*** ∨ ***Q*** |
| --- | --- |
| T T | T |
| T F | T |
| F T | T |
| F F | F |

- This table is a *characteristic truth-table* for disjunction; it defines the use of ‘∨’.
- Both ‘&’ and ‘∨’ are *binary connectives*.

# 2.2 – Introduction to Symbolization

- In SL we abbreviate 'not' by '~',

e.g., "It is not the case that it is raining" is symbolized as ~R.

- '~' is a *unary connective*.
- A sentence of the form ~***P***, where **P** is a sentence of SL, is a negation.

# 2.2 – Introduction to Symbolization

- A negation is true iff the original sentence is false, and false iff the original sentence is true:

| ***P*** | ***~P*** |
| --- | --- |
| T | F |
| F | T |

- This table is a *characteristic truth-table* for negation; it defines the use of '~' .

- Some sentential connectives can be combined to form new connectives.

# 2.2 – Introduction to Symbolization

- Truth conditions for ‘neither…nor’: true when both components are false:

| $P$ $Q$ | $\sim P \,\&\, \sim Q$ | $\sim(P \vee Q)$ |
|---|---|---|
| T T | F | F |
| T F | F | F |
| F T | F | F |
| F F | T | T |

# 2.2 – Introduction to Symbolization

- Truth conditions for ‘not both…and’ : false when both conjuncts are true:

| $P$ $Q$ | ~($P$ & $Q$) | ~$P$ ∨ ~Q |
|---|---|---|
| T T | F | F |
| T F | T | T |
| F T | T | T |
| F F | T | T |

# 2.2 – Introduction to Symbolization

- Truth conditions for exclusive 'or' : false when both components have the same truth-value:

| $P$ | $Q$ | $(P \vee Q)$ & $\sim(\mathbf{P}$ & $\mathbf{Q})$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

# 2.2 – Introduction to Symbolization

- In SL we abbreviate 'if…then' by '⊃',
- e.g., "If it rains, I will bring an umbrella" is symbolized as $R \supset U$.
- A sentence of the form $\boldsymbol{P} \supset \boldsymbol{Q}$, where $\boldsymbol{P}$ is the *antecedent* and $\boldsymbol{Q}$ is the *consequent*, is called a *material conditional*.

# 2.2 – Introduction to Symbolization

- A material conditional is false when the antecedent is true and the consequent is false:

| ***P Q*** | ***P ⊃ Q*** |
|---|---|
| T T | T |
| T F | F |
| F T | T |
| F F | T |

- This is the characteristic truth-table for the material conditional.

# 2.2 – Introduction to Symbolization

- Note: we could also express $\mathbf{P} \supset \mathbf{Q}$ as $\mathbf{\sim P} \vee \mathbf{Q}$:

| **P Q** | $\mathbf{\sim P \vee Q}$ |
|---|---|
| T T | T |
| T F | F |
| F T | T |
| F F | T |

- *Note*: ***P** only if* **Q** is symbolized as $P \supset Q$;

# 2.2 – Introduction to Symbolization

- In SL we abbreviate 'iff' by '≡',
- e.g., "I will bring an umbrella iff it rains" is symbolized as U ≡ R.
- A material biconditional is true iff P and Q have the same truth-value:

| *P Q* | *P ≡ Q* |
|---|---|
| T T | T |
| T F | F |
| F T | F |
| F F | T |

- Characteristic truth-table for the material biconditional.
- Note: we could also express **P** ≡ **Q** as (**P** ⊃ **Q**) & (**Q** ⊃ **P**) or

as (**P** & **Q**) ∨ (~**P** & ~**Q**).

# 2.3 – More Complex Symbolizations

- Each sentence of a paraphrase will be an atomic sentence, a truth-functionally compound sentence, or a non-truth-functional compound sentence.
- The atomic sentences and the non-truth-functional compound sentences are to be symbolized as atomic sentences of SL; e.g., ‘Grass is green’ as G.
- The truth-functionally compound sentences are to be symbolized as using the sentential connectives.

# 2.3 – More Complex Symbolizations

1) The British will win if neither of the other two competitors (Americans and Canadians) wins.

2) The British will win only if neither of the other two competitors wins.

1a. If (both it is not the case that A wins and it is not the case that C wins), then B will win.

2a. If B wins, then (both it is not the case that A will win and it is not the case that C will win).

1b. $(\sim A \mathbin{\&} \sim C) \supset B$

2b. $B \supset (\sim A \mathbin{\&} \sim C)$

# 2.3 – More Complex Symbolizations

- Summary of Some Common Connectives

| English Connective | English Paraphrase | Symbolization in SL |
|---|---|---|
| not **p** | it is not the case that **p** | **~P** |
| **p** and **q**<br>**p** but **q**<br>**p** however **q**<br>**p** although **q**<br>**p** nevertheless **q**<br>**p** nonetheless **q**<br>**p** moreover **q** | **p** and **q** | **P&Q** |
| **p** or **q**<br>**p** unless **q** | **p** or **q** | **PvQ** |
| **p** or **q** (but not both) | **p** or **q** and its not the case **p** and **q** | **(PvQ)&~(P&Q)** |
| If **p** then **q**<br>**q** if **p**<br>**q** provided **p**<br>**q** given **p**<br>**p** only if **q** | if **p** then **q** | **P ⊃Q** |
| **p** if and only if **q**<br>**p** if but only if **q**<br>**p** just in case **q** | **p** if and only if **q** | **P≡Q** |

# 2.4 – Non-Truth-Functional Connectives

- Many sentential connectives of English are not truth-functional.
- If the connective is being used truth-functionally, one should be able to construct a truth table that characterizes that use.
  - E.g., "If German u-boats were able to shut off the flow of supplies to Great Britain, then Germany would have won the war".
- The antecedent is false, so the conditional is true, but historians do not agree that this is true. Such *subjunctive conditionals* are not truth-functional.
- We will weaken such non-truth-functional sentences to form a truth-functional sentences or we will represent them as atomic formulas.
- In any case, we will assume that all of our sentences are truth-functional.