

May's Maxims



1. Keep up with all work!
2. To do logic you have to think logically.
Don't overthink!
3. Follow the rules!



Chapter 1

Introduction to Deductive Logic

1.1 – Introduction



- ◆ This course is about *deductive logic*, or formal or symbolic deductive logic.
- ◆ *Sentential logic*: this is the branch of symbolic deductive logic that takes *sentences* as the fundamental unit of logical analysis. (Sometimes called *propositional logic*.)
- ◆ *Predicate logic*: this is the branch of symbolic deductive logic that takes *predicates* and *individual terms* to be the fundamental units of logical analysis.

1.1 – Introduction



◆ Two concerns that drive research:

- i) Desire to formulate principles of good reasoning
 - ✦ Why are some arguments OK but others not?
 - ✦ What are mistakes of reasoning?
- ii) Desire to formalize and systematize existing/emerging work in mathematics and science.

The hallmark of deductive logic is *truth preservation*: if we start from truth and use “logical rules” we will always arrive at truths.

1.1 – Introduction



- ◆ Aristotle's syllogistic formalization of principles of good reasoning, e.g.:

All mammals are vertebrates.

Some sea creatures are mammals.

Some sea creatures are vertebrates.

All M are V

Some S are M

Some S are V

1.1 – Introduction



- ◆ This syllogism is an example of good reasoning because it is *truth preserving*.
- ◆ If the first two sentences (the *premises*) are true (no matter what **p**, **q**, and **r** are), then the third sentence (the *conclusion*) *must* be true. In other words, the form

All **p** are **q**
Some **r** are **p**
Some **r** are **q** is valid.
- ◆ In deductive logic, reasoning that is truth-preserving is said to be *valid*.

1.1 – Introduction



◆ An Invalid form:

Some **p** are **q**

All **r** are **p**

All **r** are **q**

e.g.,

Some dogs(**p**) are black(**q**)

All terriers(**r**) are dogs(**p**)

All terriers(**r**) are black(**q**)

◆ *Any* argument of this form is *invalid*.

1.1 – Introduction



- ◆ An argument form is invalid if it has *at least* one instance with true premises and a false conclusion.
- ◆ An argument can have true premises and a true conclusion *but still be invalid*.

Some birds(**p**) are hawks(**q**)

All osprey(**r**) are birds(**p**)

All osprey(**r**) are hawks(**q**)

- ◆ This argument is invalid. It is an instance of an invalid argument form.

1.1 – Introduction



◆ Problems with Aristotelian logic:

1. Many arguments have more than three sentences.
2. Every sentence must be of the form

All **p** are **q**

Some **p** are **q**

No **p** are **q**

Some **p** are not **q**

1.1 – Introduction



3. It cannot easily deal with sentences about individuals:

e.g., ‘Socrates is human’ means ‘All things that are Socrates [assume only one] are things that are human’ .

4. It cannot readily accommodate relations:

e.g., Sarah is taller than Tom and Tom is taller than Betty; therefore, Sarah is taller than Betty.

1.1 – Introduction



- Modern deductive logic
 - Emerges from mathematics in the late 19th and early 20th-century
 - Major figures are Gottlob Frege, David Hilbert, Giuseppe Peano and Bertrand Russell
 - Later in the course we will be looking at a system of natural deduction
 - ✦ Developed from ideas of Gerhard Gentzen in the 1930s.

1.1 – Introduction



- ◆ *SL*: development of systems of *sentential logic*, systems based on the way sentences of natural languages can be generated from other sentences by the use of *connectives*, such as ‘and’, ‘or’, ‘not’.
- ◆ Problems with *SL*: It cannot easily deal with reasoning that rests on claims involving *quantifiers*, such as ‘all’, ‘some’, and ‘no’.
- ◆ *PL*: *predicate logic* incorporates both *SL* and quantifiers by beginning with relations among individuals.

1.1 – Introduction



Why Study Logic?

- ◆ A discipline in its own right.
- ◆ To systematize other disciplines, e.g., geometry
- ◆ To evaluate and present arguments
- ◆ To investigate the structure and complexities of English
- ◆ An exercise in abstract thinking

1.2 Core Concepts of Deductive Logic



- ◆ Sentences are either true or false.
- ◆ Sentences have a *truth-value*: truth-value T if true, truth-value F if false.
 - Canada is located in South America
 - Has truth-value F
 - The Montreal Canadiens will win the Stanley cup
 - Time will tell whether it has truth-value T or F
- ◆ Not all sentences have truth-values, however, the formal systems we develop are intended to deal *only* with sentences that have a truth-value.
 - Lock the door when you leave has no truth-value

1.2 Core Concepts of Deductive Logic



- ◆ *****NOTE*****: we use \mathbf{p} , \mathbf{q} , \mathbf{r} as variables for sentences of *English* and use \mathbf{P} , \mathbf{Q} , \mathbf{R} as metavariables for sentences of *SL*.
- ◆ When we talk about sentences of English *in English* we will use variable variables \mathbf{p} , \mathbf{q} , \mathbf{r} , etc.
 - e.g., If \mathbf{p} is a sentence of English in SL and \mathbf{q} is a sentence of English in SL then \mathbf{p} and \mathbf{q} is also a sentence of English in SL.

1.2 Core Concepts of Deductive Logic



Defn: an *argument* is a set of two or more sentences, one of which is designated as the *conclusion* and the others are the *premises*.

- ◆ A set is an abstract object that has elements and we take sentences to be those elements.
- ◆ Standard argument form:

Premise₁

Premise₂

Premise_n

Conclusion

- ◆ *conclusion indicators*: “therefore”, “it follows that”, “thus”, “so”, “hence”, “consequently”, “as a result”
- ◆ *premise indicators*: “since”, “for”, “because”, “on account of”, “in as much as”, “for the reason that”

1.2 Core Concepts of Deductive Logic



- ◆ Discourse in English is not always in standard form

“Michael will not get the job, for whoever gets the job will have strong references, and Michael’s references are not strong.”

Whoever gets the job will have strong references

Michael’s references are not strong

Michael will not get the job

1.2 Core Concepts of Deductive Logic



- ◆ Not every sequence of sentences is an argument.
 - ◆ Sometimes they are *just* a sequence of sentences.

“Languages are very complex. They are made up of an infinite number of sentences. Each sentence is made up of words in a specified order. There are many languages in the world, and some of them are dying.”

1.2 Core Concepts of Deductive Logic



- ◆ A good, or *valid*, deductive argument is one whose form or structure is fully truth-preserving, i.e., is such that instances of it never proceed from true premises to a false conclusion.

Defn: An argument is *logically valid* if and only if it is *not possible* for the premises to be true and the conclusion false

Defn: An argument is *logically invalid* if and only if it is not logically valid.

1.2 Core Concepts of Deductive Logic



- ◆ *Note*: validity says *if* the premises are true, then the conclusion *must* also be true; it does not say the premises *are* true. E.g. the following is valid:

Clinton never lies.

Monica Trump never lies.

Therefore, Clinton never lies and ~~Monica Trump never lies~~

- ◆ Validity depends on what *may* be true, *not* what is true.

1.2 Core Concepts of Deductive Logic



- ◆ If an argument is valid, then if a person accepts that the premises are true, they *must* also accept that the conclusion is true.
- ◆ If an argument is valid, then if a person rejects that the conclusion is true, they must also reject that at least one of the premises is true.
- ◆ A person who does not do this is *irrational*.

1.2 Core Concepts of Deductive Logic



- *Defn*: An argument is *logically sound* iff it is logically valid and all its premises are true.
- *Defn* An argument is *logically unsound* iff it is not logically sound.
- ◆ Soundness requires that the premises *and* the conclusion *are* true.
- ◆ Truth is necessary but not sufficient for soundness
- ◆ Soundness also requires validity

1.2 Core Concepts of Deductive Logic



◆ *Note:* true premises and a true conclusion do not guarantee soundness:

Dogs bark

Cats meow

Therefore, Swans are white

is an argument with true premises and a true conclusion, but it is invalid, i.e., it is *possible* for the premises to be true and the conclusion false.

1.2 Core Concepts of Deductive Logic



- ◆ Every sentence that has a truth-value is either logically true, logically false, or logically indeterminate
- ◆ ***Note***: your book has these definitions in terms of \mathbf{p} , \mathbf{q} ,...because we have not yet introduced the logical language of SL.

Defn: A sentence is *logically true* iff it is not possible for the sentence to be false. E.g., and sentence of the form ' \mathbf{p} or not \mathbf{p} '

Defn: A sentence is *logically false* iff it is not possible for the sentence to be true, E.g., any sentence of the form ' \mathbf{p} and not \mathbf{p} '

Defn: A sentence is *logically indeterminate* iff it is neither logically true nor logically false. E.g., any sentence of the form ' \mathbf{p} and \mathbf{q} '

Defn: Sentences \mathbf{p} and \mathbf{q} are *logically equivalent* iff it is not possible for one of the sentences to be true while the other is false. E.g., and sentence of the form ' $\text{not}(\mathbf{p}$ and $\mathbf{q})$ and $(\text{not } \mathbf{p}$ or not $\mathbf{q})$ '.

1.2 Core Concepts of Deductive Logic



- *Defn*: A set of sentences is *logically consistent* iff it is possible for all the members of that set to be true.
- *Defn*: A set of sentences is *logically inconsistent* iff it is not logically consistent.

$\{s_1, s_2, \dots, s_k\}$ is logically consistent *iff* it is possible $s_i = T$ for all i

$\{s_1, s_2, \dots, s_k\}$ is logically inconsistent *iff* it is not possible $s_i = T$ for some i

- A set of sentences is logically inconsistent if at least one sentence *must* be false when all the others are true.

1.2 Core Concepts of Deductive Logic



- ◆ Logic cannot normally tell us whether a given sentence is true or false, but we can use logic to discover if a set of sentences is consistent or inconsistent.
- ◆ If it is consistent, then we know that it is possible that all the sentences are true.
- ◆ If it is inconsistent, then we know that at least one of the sentences in the set is false.
- *Defn*: a set of sentences *logically entails* a sentence iff it is impossible for all the members of the set to be true and that sentence to be false.

1.3 – Special Cases of Logical Concepts



- ◆ Any argument with a logically true conclusion is valid.
- ◆ Every logically true sentence is entailed by every set of sentences.
- ◆ Any argument with an inconsistent set of premises is valid.

- ◆ All logically true sentences are equivalent
- ◆ All logically false sentences are equivalent